Question Paper - Report

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Question Paper

MANIPAL ACADEMY OF HIGHER EDUCATION

COMPUTATIONAL MATHEMATICS - I [MAT 1172]

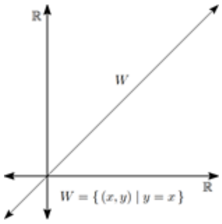
Marks: 30 **Duration: 90 mins.**

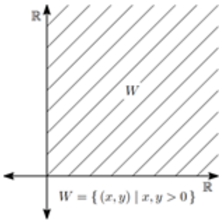
A

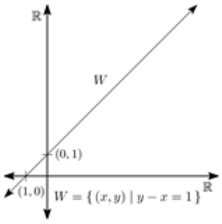
Answer all the questions. Section Duration: 20 mins

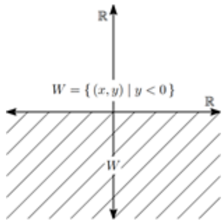
Select the correct answer for the following questions

1) Which of the following is a subspace of $\mathbb{R}^2$? (0.5)









2) Let $A = [a_{ij}]$ be a 2×4 matrix having rank 1. If the first row of the matrix A is [1 5 -2 4] and $a_{24} = 12$ then $a_{22} =$ (0.5)

20 5 15 0

3) If A is an $n \times n$ matrix with rank n, then its row reduced echelon form is (0.5)

a singular matrix. the identity matrix of order *n*. a symmetric matrix. a skew-symmetric matrix.

4) The rank of the matrix $\begin{bmatrix} -1 & 2 & -3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix}$ is (0.5)

3 2 1 0

5) Let A= $\begin{bmatrix} x & 1 \\ 1 & 2 \end{bmatrix}$. Then, the value of x for which 0 is an eigevalue of A is (0.5)

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$

6) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then eigen values of A^2 are (0.5)

$1, 3, 2$ $1, 0.5, 0.33$ $1, 4, 9$ $1, 2, -3$

7) Using Gauss Jacobi's method, the first approximate value of y from the system of linear equations, (0.5)

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

$$20x + y - 2z = 17$$

by taking the initial approximation as $(x, y, z) = (0,0,0)$ is

-0.9 -8.3333 0.09 0.85

8) Let $S = \{(-1,2),(2,3)\}$ be a subset of $\mathbb{R}^2$. Then the linear span of S is $L(S) =$.

(0.5)

$\{(\alpha-2\beta,\alpha-3\beta) \mid \alpha,\beta \in \mathbb{R}\}$ $\{(\alpha+2\beta,2\alpha-3\beta) \mid \alpha,\beta \in \mathbb{R}\}$ $\{(-\alpha+2\beta,2\alpha-3\beta) \mid \alpha,\beta \in \mathbb{R}\}$ $\{(-\alpha+2\beta,2\alpha+3\beta) \mid \alpha,\beta \in \mathbb{R}\}$

9) The system of linear equations $AX = B$ with m unknowns can have infinitely many solutions if

(0.5)

$\rho(A) = \rho[A:B] = m$ $\rho(A) \neq \rho[A:B]$ Echelon form of A is an identity matrix $\rho(A) = \rho[A:B] < m$

10) Let T be a linear operator on $\mathbb{R}^2$ such that $T(1,0) = (1,1)$ and $T(1,1) = (1,0)$. Then the matrix representation of T with respect to the standard basis $\{(1,0),(0,1)\}$ is

(0.5)

$\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$

B

Answer all the questions.

Answer all the questions.Any missing data can be assumed suitably with proper reasoning.

11) Diagonalize the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ (4)

12) Test whether the set of vectors in $\mathbb{R}^3$ form a basis or not. If so, express the vector $(4,5,3)$ in terms of basis vectors. (3)

13) Prove that the orthogonal set $S = \{v_1, v_2, \dots, v_n\}$ of non-zero vectors in an inner product space V is linearly independent. (3)

14) Using Gauss elimination method, test the consistency and solve the system of linear equations (3)

$$x + y + z = 8$$

$$x - y + 2z = 6$$

$$3x + 5y - 7z = 14$$

15) Using Gauss Seidel method, find the approximate solution of the system of linear equations (3)

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20$$

Take $x^{(0)} = y^{(0)} = z^{(0)} = 0$. Carry out three iterations and correct to three decimal places.

16) Using Gram-Schmidt orthogonalization process, find an orthonormal basis of $\mathbb{R}^3$ from the linearly independent set of vectors (3)

$$S = \{a_1 = (1,2,2), a_2 = (-1,0,2), a_3 = (0,0,1)\}.$$

17) Test whether the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by (2)

$T(x,y) = (2x + y, 3y)$ for all $(x,y) \in \mathbb{R}^2$

is a linear transformation or not.

18) (2)

Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with $T(1,0) = (1,1,1)$ and $T(0,1) = (1,-1,1)$. Find the matrix representation of T with respect to the basis $\{(1,0),(0,1)\}$ of $\mathbb{R}^2$ and $\{(1,1,0),(1,-1,0),(0,0,1)\}$ of $\mathbb{R}^3$.

19) Solve the differential equation

$$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$$ (2)

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