

Question Paper

**MANIPAL ACADEMY OF HIGHER EDUCATION****COMPUTATIONAL MATHEMATICS - II [MAT 1272]****Marks: 30****Duration: 90 mins.****MCQ****Answer all the questions.**

Section Duration: 20 mins

- 1) The degree of homogeneous function

$$f(x, y) = \sqrt{\frac{\frac{1}{x^2} + \frac{1}{y^2}}{\frac{1}{x^3} + \frac{1}{y^3}}}$$

is ____

(0.5)

$$\frac{1}{12} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{2}$$

- 2) The percentage error in the area of a rectangle when an error of 1% is made in measuring its length and breadth is ____

(0.5)

$$\underline{2\%} \quad \underline{1\%} \quad \underline{0\%} \quad \underline{3\%}$$

- 3) The coefficient of
- y^3
- in the Maclaurin's series expansion of
- $xy + \cos y$
- is ____

(0.5)

$$\underline{2} \quad \underline{0} \quad \underline{4} \quad \underline{6}$$

- 4) If
- $u = f(r, s)$
- and
- $r = x + at$
- ,
- $s = y + bt$
- , where

 a
and b

are constants, then

(0.5)

$$\frac{\partial u}{\partial t} =$$

$$a \frac{\partial r}{\partial x} a \frac{\partial r}{\partial x} + b \frac{\partial s}{\partial y} a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

- 5) The value of

$$\frac{\Gamma(7/2)}{\Gamma(5/2)}$$

is

(0.5)

$$\underline{3.5} \quad \underline{4.5} \quad \underline{2.5} \quad \underline{1.4}$$

- 6) The value of

$$\int_0^1 x^4 (1 - \sqrt{x})^3 dx$$

is

(0.5)

$$2\beta(10, 4) \quad \beta(10, 4) \quad \beta(9, 3) \quad 2\beta(6, 4)$$

- 7)

(0.5)

On changing to spherical polar coordinates, the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$$

is ____

$$\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \frac{r^2 \sin \theta}{\sqrt{1-r^2}} dr d\theta d\phi \int_0^{2\pi} \int_0^{\pi} \int_0^1 \frac{r^2 \sin \theta}{\sqrt{1-r^2}} dr d\theta d\phi \int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \frac{r^2 \sin \theta}{\sqrt{1-r^2}} dr d\theta d\phi \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 \frac{r^2 \sin \theta}{\sqrt{1-r^2}} dr d\theta d\phi$$

8) The value of the double integral

$$\iint_R \frac{1}{\sqrt{x^2+y^2}} dx dy$$

, where

R

is the region bounded by the circle

$$x^2 + y^2 = 1$$

is

(0.5)

$$\pi \quad 2\pi \quad \frac{\pi}{2} \quad 0$$

9) The value of

$$\int_{x=1}^2 \int_{y=2}^3 \int_{z=1}^3 (x^2 + 1) dz dy dx$$

is

(0.5)

$$\frac{10}{3} \quad \frac{20}{3} \quad \frac{22}{3} \quad \frac{20}{7}$$

10) If

$$u = x + y, \quad v = 3x - 2y,$$

then $\frac{\partial(x, y)}{\partial(u, v)}$ is ____

(0.5)

$$-\frac{1}{5} \quad -5 \quad -1 \quad \underline{1}$$

DESCRIPTIVE

Answer all the questions.

11) Using triple integral, find the volume of the portion of sphere $x^2 + y^2 + z^2 = 9$ lying inside the cylinder $x^2 + y^2 = 3y$ above the xy -plane. (4)

12) Expand $f(x, y) = \tan^{-1}(xy)$ about the point $(1, 1)$ up to second degree terms. (3)

13) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$. (3)

14) If $u = \sin^{-1}\left(\frac{x^2 y^2}{x+y}\right)$, then prove that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 3 \tan u (3 \tan^2 u + 2)$. (3)

15) Calculate the line of regression for the following data:

x	0	25	50	75	100
y	14	38	54	76	95

(3)

16) Evaluate $\int_0^1 \int_y^{2-y} xy \, dx dy$ by changing the order of integration. (3)

17) Evaluate $\int_0^1 \int_0^{1-x} (y-x)^2 \sqrt{x+y} \, dy dx$, using the transformations $x + y = u, y - x = v$. (3)

18) Use double integration to find the area of the region inside the circle $r = 4\cos\theta$ and outside the circle $r = 2$. (3)