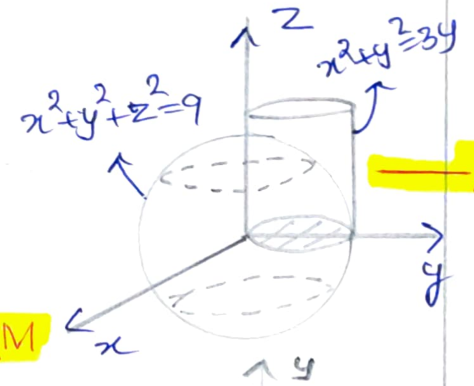
Midterm Examination - March 2025
Computational Mathematics - II [MAT 1272]

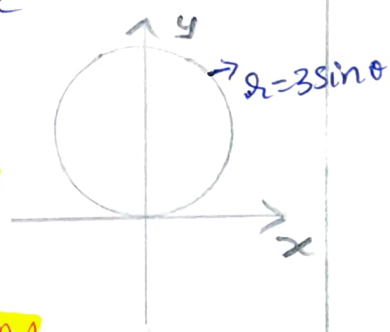
11 Using Cylindrical co-ordinates,

$$V = 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{3\sin\theta}\int_{z=0}^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta \text{ ——— } 1\tfrac{1}{2}M$$



— ½M

$$= 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{3\sin\theta} r\sqrt{9-r^2}\,dr\,d\theta \text{ ——— } \tfrac{1}{2}M$$



$$= 2\int_{\theta=0}^{\pi/2} -\tfrac{1}{3}(9-r^2)^{3/2}\Big|_{r=0}^{3\sin\theta}\,d\theta \text{ ——— } \tfrac{1}{2}M$$

$$= -\tfrac{2}{3}\int_0^{\pi/2}(\cos^3\theta - 27)\,d\theta \text{ ——— } \tfrac{1}{2}M$$

$$= \underline{\underline{9\pi - 12}} \text{ ——— } \tfrac{1}{2}M$$

12 $f(x,y) = \tan^{-1}(xy)$

$$f_x = \frac{1}{1+x^2y^2}\times y \;;\quad f_y = \frac{x}{1+x^2y^2} \text{ ——— } 1M$$

$$f_{xx} = \frac{-2xy^3}{(1+x^2y^2)^2} \;;\quad f_{xy} = \frac{1-x^2y^2}{1+x^2y^2} \;;\quad f_{yy} = \frac{-2x^3y}{(1+x^2y^2)^2} \text{ ——— } 1M$$

$$\left.\begin{array}{l} f(1,1) = \pi/4 ,\quad f_x(1,1) = 1/2 ,\quad f_y(1,1) = 1/2 \\ f_{xx}(1,1) = -1/2 \;;\quad f_{yy}(1,1) = -1/2 \;;\quad f_{xy}(1,1) = 0 \end{array}\right\} \tfrac{1}{2}M$$

$$\tan^{-1}(xy) = \frac{\pi}{4} + \frac{1}{2}(x+y-2) - \frac{1}{2\times 2}\left((x-1)^2 + (y-1)^2\right) + \cdots \text{ ——— } \tfrac{1}{2}M$$

13 $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$

$f_x = 0 \Rightarrow 3x^2 - 63 + 12y = 0$

$f_y = 0 \Rightarrow 3y^2 - 63 + 12x = 0$

Stationary points: $(3,3)\ (-7,-7)\ (-1,5)\ (5,-1)$ — 1½ M

$A = f_{xx} = 6x, \quad B = f_{xy} = 12, \quad C = f_{yy} = 6y$ — ½ M

Stationary Points	A	$AC-B^2$	
(3, 3)	18	180	→ Min
(−7, −7)	−42	1620	→ Max
(5, −1)	30	−324	
(−1, 5)	−6	−324	

Maximum value = 784

Minimum value = −216 — 1M

14 $u = \sin^{-1}\left(\dfrac{x^2y^2}{x+y}\right)$

Let $z = \sin u = \dfrac{x^2y^2}{x+y}$ → is homogeneous function of degree 3 — ½ M

By Euler's theorem,

$x z_x + y z_y = 3z$ — ½ M

$x u_x + y u_y = 3\tan u$ —(i)

Diff (i) partially w.r.to x

$x u_{xx} + u_x + y u_{xy} = 3(\sec^2 u) u_x$

$\times^{ing}$ by x $\quad x^2 u_{xx} + xy u_{xy} = [3\sec^2 u - 1] x u_x$ —(ii) — $\frac{1}{2}$M

Similarly, Differentiate (i) partially w.r.to y & multiplying by 'y'

$$y^2 u_{yy} + xy\, u_{xy} = [3\sec^2 u - 1]\, y\, u_y \quad \text{--- (iii)}$$ ½M

(ii)+(iii) ⇒

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = [3\sec^2 u - 1]\, 3\tan u$$
$$= [3(1+\tan^2 u) - 1]\, 3\tan u$$
$$= 3\tan u\,[3\tan^2 u + 2]$$

1M

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$
0	14	-50	-41.4	2070	2500
25	38	-25	-17.4	435	625
50	54	0	-1.4	0	0
75	76	25	20.6	515	625
100	95	50	39.6	1980	2500

1½M

15 $\bar{x} = \frac{250}{5} = 50$; $\bar{y} = \frac{277}{5} = 55.4$ ½M

Slope, $\beta_1 = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \frac{5000}{6250} = 0.8$ ½M

Intercept, $\beta_0 = \bar{y} - \beta_1 \bar{x} = 55.4 - 0.8(50) = 15.4$

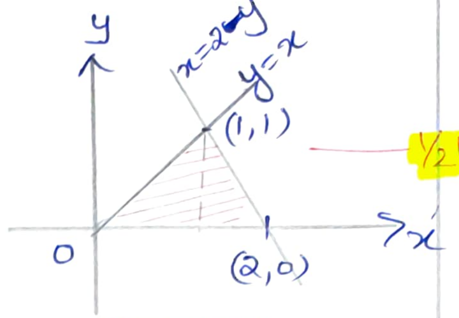
½M

$\therefore y = 0.8x + 15.4$

16

$$\int_0^1 \int_y^{2-y} xy \, dx\, dy$$

$$= \int_{x=0}^{1} \int_{y=0}^{x} xy \, dy\, dx + \int_{x=1}^{2} \int_{y=0}^{2-x} xy \, dy\, dx$$



½ M

(1+1) M

$$= \int_{x=0}^{1} \frac{x^3}{2} dx + \int_1^2 \frac{x(2-x)^2}{2} dx = \frac{1}{3}$$ ½ M

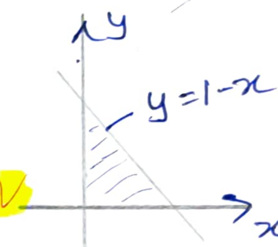
17

$x+y=u, \quad y-x=v \Rightarrow x = \frac{u-v}{2}$

$y = \frac{u+v}{2}$

$J_1 = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2 \quad \& \quad J = \frac{1}{2}$

1M

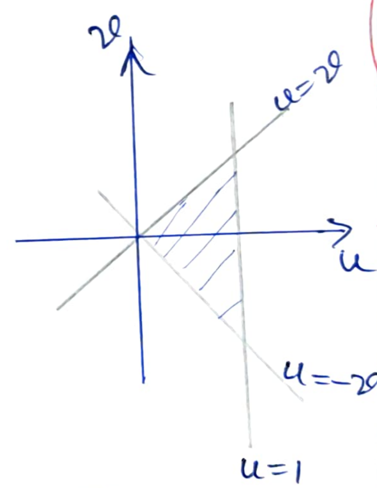


when $x=0$, $u=v$

when $y=0$, $u=-v$

when $x+y=1$, $u=1$

1M

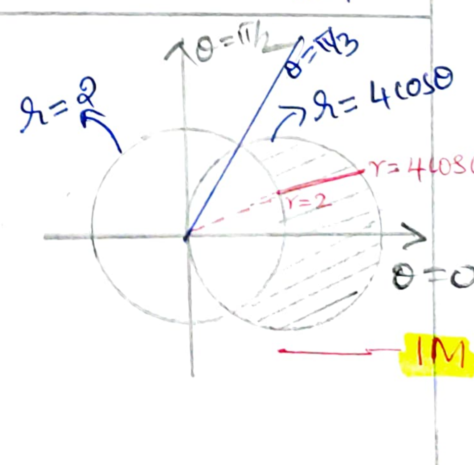


$$I = \int_{u=0}^{1} \int_{v=-u}^{u} v^2 \sqrt{u} \left(\tfrac{1}{2}\right) dv\, du$$

$$= \frac{1}{3} \int_{u=0}^{1} u^{7/2} du = \frac{2}{27}$$ 1M

18.

$$A = 2 \int_{\theta=0}^{\pi/3} \int_{r=2}^{4\cos\theta} r \, dr\, d\theta$$ 1M



1M

$$= \int_0^{\pi/3} (16\cos^2\theta - 4) \, d\theta$$ ½ M

$$= \frac{4\pi}{3} + 2\sqrt{3}$$ ½ M