SUBJECT: ENGINEERING MATHEMATICS-I [MAT 1151] (CHEMISTRY CYCLE)
SCHEME OF EVALUATION

Q. No.	DESCRIPTION	MARKS
1A **Solution**	**Solve** $\boldsymbol{y - \cos x\frac{dy}{dx} = y^2(1 - \sin x)\cos x}$ **given** $\boldsymbol{y(0) = 2.}$ To solve: $-cosx\frac{dy}{dx} = y^2(1 - sinx)cosx - y$ Divide by y^2cosx, Then, $\frac{-1}{y^2}\frac{dy}{dx} + \frac{1}{y}\sec x = 1 - sinx$ Substitute $\frac{1}{y} = z$, then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$ The above DE, $\frac{dz}{dx} + z\,secx = 1 - sinx$ which is linear in z. I.F = $e^{\int secxdx} = e^{\log(secx+tanx)} = secx + tanx$ The general solution in z is, $z(secx + tanx) = \int(1 - sinx)(secx + tanx)dx$ By back substituting y, $\frac{(secx+tanx)}{y} = \int(1 - sinx)\frac{(1+sinx)}{cosx}dx + c$ By simplifying, $\frac{(secx+tanx)}{y} = \int cosx\,dx + c$ i.e.,w $\frac{(secx+tanx)}{y} = sinx + c$ By substituting the initial condition, $y(0) = 2$, the value of $c = 0.5$. The particular solution is, $secx + tanx = ysinx + \frac{y}{2}$	 **1** **1** **1** **1**
1B **Solution**	**Solve** $\boldsymbol{(2x - 1)^3y''' + (2x - 1)y' - 2y = 8x^2 - 2x + 3.}$ By substituting, $(2x - 1) = e^t$. Then, $x = \frac{e^t+1}{2}$ and $t = \log(2x - 1)$. The above DE, $(2^3D(D - 1)(D - 2) + 2D - 2)y = 2e^{2t} + 3e^t + 4$ A.E, $8m^3 - 24m^2 + 18m - 2 = 0$ Roots are 1, $\left(1 \pm \frac{\sqrt{3}}{2}\right)$ The complementary function is, $y_c = c_1e^t + c_2e^{(1+\frac{\sqrt{3}}{2})t} + c_3e^{(1-\frac{\sqrt{3}}{2})t}$ The particular integral, $y_p = \frac{1}{8D^3-24D^2+18D-2}(2e^{2t} + 3e^t + 4)$ i.e, $y_p = \frac{2e^{2t}}{2} + \frac{3te^t}{18} - \frac{4}{2}$ Therefore, $y = c_1(2x - 1) + c_2(2x - 1)^{\left(1+\frac{\sqrt{3}}{2}\right)} + c_3(2x - 1)^{\left(1-\frac{\sqrt{3}}{2}\right)} +$ $(2x - 1)^2 + \frac{(2x-1)log(2x-1)}{6} - 2.$	**1.5** **0.5** **1** **0.5** **1.5** **1**

2A

Using Lagrange's interpolation formula find $y(7)$ from the following table.

x	5	6	9	11
y	12	13	14	16

Solution

We have, By Lagrange's interpolation formula,

$x = 7, x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11, y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16.$

From the formula, $y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$ **1**

$$= \frac{(7-6)(7-9)(7-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(7-5)(7-9)(7-11)}{(6-5)(6-9)(6-11)}(13) + \frac{(7-5)(7-6)(7-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(7-5)(7-6)(7-9)}{(11-5)(11-6)(11-9)}(16)$$ **3**

$$= -\frac{96}{24} + \frac{208}{15} + \frac{112}{24} - \frac{64}{60} = 13.4667$$

2B

Solve the initial value problem, $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0) = 1$ at $x = 0.2$ by using Modified Euler's method, take $h = 0.1$. Modify the solution twice in each step. Use this value $y(0.2)$ and find y at $x = 0.3$ using Runge - Kutta method of order 4 with $h = 0.1$.

Solution

To find $y(0.2)$ using Modified Euler's method, $h = 0.1$

$x_0 = 0, \; x_1 = 0.1, \; x_2 = 0.2$

$$y_1^{(0)} = 1.1$$
$$y_1^{(1)} = 1.0991535$$
$$y_1^{(2)} = 1.099179$$

Thus, $y(0.1) = 1.099179$ **1.5**

$$y_2^{(0)} = 1.197537$$
$$y_2^{(1)} = 1.1950115$$
$$y_2^{(2)} = 1.1956333$$

Thus, $y(0.2)$= 1.1956333 **1.5**

R-K method to find $y(0.3)$:

$k_1 = 0.094556,$ **0.5**

$k_2 = 0.092227,$ **0.5**

$k_3 = 0.09221,$ **0.5**

$k_4 = 0.08971,$ **0.5**

Hence, $y(0.3) = 1.28779$ **1**

3A **The following table gives the velocity of a particle at time t.**

$t(sec)$	0	2	4	6	8	10	12
v (m/sec)	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 sec and the acceleration at $t = 2\ sec$.

Solution

We know $\frac{ds}{dt} = v, \frac{dv}{dt} = a$

Where v = velocity; a = acceleration and s = distance

$$\therefore ds = v\,dt \Rightarrow s = \int v\,dt$$

$\therefore$ The distance moved by the particle in 12 sec $= \int_0^{12} v\,dt$. **1**

Here $h = 2$. By Simpson's $\frac{1}{3}$ rule,

$$\therefore \int_0^{12} v\,dt = \frac{h}{3}[(v_0 + v_6) + 4\,(v_1 + v_3 + v_5) + 2(v_2 + v_4)]$$

$$= \frac{2}{3}[(4 + 136) + 4(6 + 34 + 94) + 2(16 + 60)] = 552$$

$\therefore$ Distance covered =552 meters

$$Acceleration = \left(\frac{dv}{dt}\right)_{t=2}$$

t	v	Δv	$\Delta^2 v$
0	4		
		2	
2	**6**		8
		10	
4	16		**8**
		18	
6	34		8
		26	
8	60		8
		34	
10	94		8
		42	
12	136		

2

Thus, acceleration is, $\left(\frac{dv}{dt}\right)_t = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \cdots\right)$

$$= \frac{1}{2}\left(10 - \frac{8}{2}\right) = 3m/\sec^2$$ **1**

<table>
<tr><td>3B

Solution</td><td>Solve the system of equations, correct to 4 decimal places by using Gauss - Seidel method:
$\boldsymbol{x + 4y + 2z = 15;\ 5x + 2y + z = 12;\ x + 2y + 5z = 20}$, Carryout 3 iterations.

Re-write the system of equations in Diagonally dominant form
$x = \frac{12-2y-z}{5}$; $y = \frac{15-x-2z}{4}$; z= $\frac{20-x-2y}{5}$

<table>
<tr><td>Iteration</td><td>x</td><td>y</td><td>z</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>2.4</td><td>3.15</td><td>2.26</td></tr>
<tr><td>2</td><td>0.688</td><td>2.448</td><td>2.8832</td></tr>
<tr><td>3</td><td>0.8442</td><td>2.0974</td><td>2.9922</td></tr>
</table>
</td><td>0.5

0.5M for each iteration</td></tr>
<tr><td>3C

Solution</td><td>Find all the eigenvalues and an eigenvector corresponding to the least eigenvalue of the matrix
$$\boldsymbol{A = \begin{bmatrix} 6 & -2 & -2 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}}$$
The characteristic equation of matrix A is $det(A - \lambda I) = 0$
$\lambda^3 - 17\lambda^2 + 90\lambda - 144 = 0$

The eigenvalues are $\lambda = 3,\ 8,\ 6$

The eigenvector corresponding to $\lambda = 3$ is $[A - 3I]X = 0$
$$\begin{bmatrix} 3 & -2 & -1 \\ -2 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$\frac{x}{5} = \frac{-y}{-5} = \frac{z}{5}$=k

Thus $X = [k,\ k,\ k]^T, k \in R$ is the eigenvector.</td><td>1.5

1

1

0.5</td></tr>
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4A **Solution**	**Find a multiple root of the equation using Newton's method, $x^3 - x^2 - 8x + 12 = 0$. Take $x_0 = 1.8$.** Newton's method for multiple roots, with $x_0 = 1.8$ $x_0 - 3\frac{f(x_0)}{f'(x_0)} = 2.10638$ $x_0 - 2\frac{f'(x_0)}{f''(x_0)} = 2.22727$ $x_0 - \frac{f''(x_0)}{f'''(x_0)} = 0.33333$ Double root at 2. Let us take x_1 =2.10638 $x_0 - 2\frac{f(x_0)}{f'(x_0)} = 2.0011$ $x_0 - \frac{f'(x_0)}{f''(x_0)} = 2.00319$ The approximate root correct to 2 decimal places is 2.	**2** **2**
4B **Solution**	**Express the vector $(1, -2, 5)$ as a linear combination of the vectors $\{(1,1,1), (1,2,3), (2,-1,1)\}$ in $\mathbb{R}^3$.** Let $(1,-2,5) = a\,(1,1,1) + b\,(1,2,3) + c(2,-1,1)$ $= (a + b + 2c, a + 2b - c, a + 3b + c)$ Equating and solving: $a = -6, b = 3, c = 2$.	**1** **1**
4C **Solution**	**Construct an orthonormal basis using Gram-Schmidt orthogonalization process from the set of linearly independent vectors $\{v_1 = (1,0,1), v_2 = (1,0,-1), v_3 = (0,3,4)\}$.** $u_1 = \frac{v_1}{\|v_1\|} = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ Now $a_2 = v_2 - (v_2.u_1)\,u_1 = (1,0,-1) - 0.\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = (1,0,-1)$ and $u_2 = \frac{a_2}{\|a_2\|} = \frac{(1,0,-1)}{\sqrt{1^2+0^2+(-1)^2}} = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ $a_3 = v_3 - (v_3.u_2)u_2 - (v_3.u_1)u_1 = (0,3,0)$ $u_3 = \frac{a_3}{\|a_3\|} = \frac{(0,3,0)}{\sqrt{0^2+3^2+0^2}} = (0,1,0)$. Hence the required set of orthonormal vectors is $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$, $(0,1,0)$	**1** **1** **1** **0.5** **0.5**