Question Paper

MANIPAL ACADEMY OF HIGHER EDUCATION

B.Tech First Semester Sessional Examination September 2024

ENGINEERING MATHEMATICS - I [MAT 1171]

Marks: 30 **Duration: 90 mins.**

MCQ

Answer all the questions. Section Duration: 20 mins

1) Using Jacobi's Method with initial approximation $x^{(0)} = y^{(0)} = z^{(0)} = 0$,

the value of x in the second iteration of the following system of equations is_____

$$2x + y + z = 5$$
$$3x + 5y + 2z = 15$$
$$2x + y + 4z = 8$$

(0.5)

0 1.5625 2.5 1.5

2) The rank of the matrix $A = \begin{bmatrix} 3 & 1 & -1 & -5 \\ 0 & 0 & 0 & -2 \\ 0 & 7 & -8 & 1 \end{bmatrix}$ is____

(0.5)

3 1 2 4

3) If two of the eigenvalues of $A = \begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$ are 0 and 2, then the eigenvalues of A^3 are ____

(0.5)

0, 2, 4 0, 8, 64 0, 4, 16 0, -2, -4

4) Which of the following set of vectors is linearly independent in $\mathbb{R}^2$?

(0.5)

$\{(0,-1),(2,0),(2,-1)\}$ $\{(-2,3),(2,-3)\}$ $\{(6,0),(0,-3)\}$ $\{(5,0),(-7,0)\}$

5)

If the eigenvector of $A=\begin{bmatrix}0 & 5 & -10\\0 & 22 & 16\\0 & -9 & -2\end{bmatrix}$ is $X=\begin{bmatrix}-5\\-4\\3\end{bmatrix}$, then the corresponding eigenvalue of A is___ (0.5)

4 5 10 19

6) The system of equation $AX = B$ with n unknowns has unique solution if and only if ____ (0.5)

$rank(A)=rank(A|B)=nrank(A)=rank(A|B)rank(A)\neq rank(A|B)rank(A)=rank(A|B)<n$

7) The value of k such that the vector $\begin{bmatrix}5\\-2\\4\end{bmatrix}$ is an eigenvector of the matrix $A=\begin{bmatrix}5 & -10 & -5\\2 & k & 2\\-4 & -8 & 6\end{bmatrix}$ is____ (0.5)

1 14 12 17

8) The integrating factor of the differential equation $\frac{dy}{dx}-3ycot\ x=\sin 2x$, to reduce it to an exact differential equation is ___ (0.5)

$\sin x$ $\text{cosec}^3 x$ $\sin^3 x$ $\text{cosec}\ x$

9) If $A=\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 0\end{bmatrix}$, then the characteristic equation of A is ____ (0.5)

$\lambda^3-3\lambda^2+2\lambda=0$ $\lambda^3+3\lambda^2+2\lambda=0$ $\lambda^3+3\lambda^2-2\lambda=0$ $\lambda^3-3\lambda^2-2\lambda=0$

10) The rank of the matrix $A=\begin{bmatrix}2 & 1 & 3\\3 & k & -1\\5 & -4 & 2\end{bmatrix}$ is 2 when k is ____ (0.5)

-5 4 -3 5

DESCRIPTIVE

Answer all the questions.

11) Test whether the set of vectors $B=\{(2,2,1),(1,3,1),(1,2,2)\}$ forms a basis for $\mathbb{R}^3$ or not. If so, express the vector (3, 1, 1) in terms of basis vectors. (4)

12) Solve the differential equation $(4xy + 3y^2 - x)dx + (x^2 + 2xy)dy = 0$ (3)

13) Find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ (3)

14) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$, using Gauss-Jordan method. (3)

15) Test for consistency, the following system of equations and hence solve by Gauss Elimination method

$$x + y + z = 3$$

$$2x - y - z = 3$$

$$x - y + z = 9$$

(3)

16) Solve the differential equation: $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$ (3)

17) Using Gauss Seidel method with initial approximation $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$, solve the following system of equations

$$-3x_1 + 22x_2 + 2x_3 = 47;$$

$$45x_1 + 2x_2 + 3x_3 = 58;$$

$$5x_1 + x_2 + 20x_3 = 67.$$

(2)

Carry out 2 iterations up to 3 decimal place accuracy.

18) Solve the differential equation

$$(3x^2 tany - cosx)dx + x^3 sec^2 y dy = 0$$

(2)

19) Using Rayleigh power method, find the numerically largest eigenvalue and the

corresponding eigenvector of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using the initial vector $X^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (2)

Carry out 2 iterations up to three decimal place accuracy.

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