Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIRST SEMESTER B. TECH (CORE BRANCHES)
MID SEMESTER EXAMINATIONS, SEPTEMBER 2024

ENGINEERING MATHEMATICS I [MAT-1171]
REVISED CREDIT SYSTEM

Date : 23.09.2024 TIME: 8.30AM-10.00AM

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

Q.NO	Questions	Marks
11	Test whether the set of vectors $B = \{(2,2,1),(1,3,1),(1,2,2)\}$ forms a basis for $\mathbb{R}^3$ *or not*. If so express the vector $(3,1,1)$ in terms of basis vectors. Soln: Consider $\lambda_1(2,2,1)+\lambda_2(1,3,1)+\lambda_3(1,2,2)=(0,0,0)$-----(1M) $\begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 5 \neq 0.$ -------(0.5M) The set $\{(2,2,1),(1,3,1),(1,2,2)\}$ is linearly independent. As there are 3 linearly independent set of vectors from $\mathbb{R}^3$, this forms a basis for $\mathbb{R}^3$.-------------(1M) Consider $\lambda_1(2,2,1)+\lambda_2(1,3,1)+\lambda_3(1,2,2)=(3,1,1)$. $2\lambda_1+\lambda_2+\lambda_3=3$ $2\lambda_1+3\lambda_2+2\lambda_2=1$ $\lambda_1+\lambda_2+2\lambda_3=1$--------(0.5M) Solving we get $\lambda_1=2, \lambda_2=-1, \lambda_3=0$ $2(2,2,1)-1(1,3,1)+0(1,2,2)=(3,1,1)$. -------(1M)	*4*
12	Solve $(4xy+3y^2-x)dx+(x^2+2xy)dy=0$ Soln: $\frac{\partial M}{\partial y}=4x+6y$ $\frac{\partial N}{\partial x}=2x+2y$ $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{2}{x}$ ---------------------(1M) Hence, IF=$e^{\int \frac{2}{x}dx}=x^2$ --------(0.5 M)	*3*

	$$x^2(4xy+3y^2-x)dx+x^2(x^2+2xy)dy=0$$ Soln: $\int x^2(4xy+3y^2-x)dx=c$ -----------(0.5 M) $x^4y+x^3y^2-\frac{x^4}{4}=c$ --------------- (1M)	
13	Find all the eigenvalues and the corresponding eigenvectors of the matrix $A=\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$. Soln: $\lvert A=\lambda I\rvert=\begin{vmatrix}5-\lambda & 4\\1 & 2-\lambda\end{vmatrix}=0\Rightarrow\lambda^2-7\lambda+6=0$ Hence eigenvalues are 1,6.----------------------(1M) Eigenvector corresponding to $\lambda=1$ is $$AX=X\Rightarrow\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$ Simplifying we get, $x=-y$. Let $x=k\neq 0$. Hence $X=\begin{bmatrix}k\\-k\end{bmatrix}, k\neq 0.$ ----------(1M) Eigenvector corresponding to $\lambda=6$ is $$AX=X\Rightarrow\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=6\begin{bmatrix}x\\y\end{bmatrix}$$ Simplifying we get, $x=4y$. Let y= $k\neq 0$. Hence $X=\begin{bmatrix}4k\\k\end{bmatrix}, k\neq 0.$ -------(1M)	
14	Find the inverse of the following matrix using Gauss-Jordan method $$A=\begin{bmatrix}1 & 0 & 2\\2 & -1 & 3\\4 & 1 & 8\end{bmatrix}.$$ Soln: $\begin{pmatrix}1 & 0 & 2 & 1 & 0 & 0\\2 & -1 & 3 & 0 & 1 & 0\\4 & 1 & 8 & 0 & 0 & 1\end{pmatrix}$---------------(0.5M) $R_2:R_2-2R_1$ and $R_3:R_3-4R_1$ $:\begin{pmatrix}1 & 0 & 2 & 1 & 0 & 0\\0 & -1 & -1 & -2 & 1 & 0\\0 & 1 & 0 & -4 & 0 & 1\end{pmatrix}$------(0.5M) $$R_3:R_3+R_2$$ $$\sim\begin{pmatrix}1 & 0 & 2 & 1 & 0 & 0\\0 & -1 & -1 & -2 & 1 & 0\\0 & 0 & -1 & -6 & 1 & 1\end{pmatrix}$$ $R_2:R_2-R_3$ $\sim\begin{pmatrix}1 & 0 & 2 & 1 & 0 & 0\\0 & -1 & 0 & 4 & 0 & -1\\0 & 0 & -1 & -6 & 1 & 1\end{pmatrix}$------(0.5M) $$R_3:-R_3$$	***3***

	$\sim\begin{pmatrix}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -4 & 0 & 1\\ 0 & 0 & 1 & 6 & -1 & -1\end{pmatrix}$-----------(0.5M) $R_1: R_1 - 2R_3$ $\begin{pmatrix}1 & 0 & 0 & -11 & 2 & 2\\ 0 & 1 & 0 & -4 & 0 & 1\\ 0 & 0 & 1 & 6 & -1 & -1\end{pmatrix}$-------(0.5M) $A^{-1} = \begin{pmatrix}-11 & 2 & 2\\ -4 & 0 & 1\\ 6 & -1 & -1\end{pmatrix}$------------(0.5M)	
15	Test for consistency and hence solve by Gauss Elimination method $x + y + z = 3$ $2x - y - z = 3$ $x - y + z = 9$ Solution: $[A\|B] = \begin{bmatrix}1 & 1 & 1 & 3\\ 2 & -1 & -1 & 3\\ 1 & -1 & 1 & 9\end{bmatrix}$--------(0.5M) $R_2: R_2 - 2R_1$ and $R_3: R_3 - R_1$ will give $\sim\begin{bmatrix}1 & 1 & 1 & 3\\ 0 & -3 & -3 & -3\\ 0 & -2 & 0 & 6\end{bmatrix}$---------(0.5M) $R_3: R_3 - \frac{2}{3}R_2$ $\sim\begin{bmatrix}1 & 1 & 1 & 3\\ 0 & -3 & -3 & -3\\ 0 & 0 & 2 & 8\end{bmatrix}$ $R_2: \frac{R_2}{-3}$ and $R_3: \frac{R_3}{2}$ $\sim\begin{bmatrix}1 & 1 & 1 & 3\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 4\end{bmatrix}$--------(0.5M) $Rank\ (A) = Rank\ (A\|B) = 3$ (unique soln) ----(0.5M) Hence by back substitution, $z = 4, y = -3$ and $x = 2$. -----(1M)	***3***
16	Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x \cos^2 x}{y^2}$ Soln: Given $y^2\frac{dy}{dx} - y^3\tan x = \sin x\cos^2 x$ - Put $y^3 = t$; $3y^2\frac{dy}{dx} = \frac{dt}{dx}$ ----------(0.5M) Then we get $\frac{dt}{dx} - 3t\tan x = 3\sin x\cos^2 x$ --------(1M) Therefore, $I.F. = e^{-\int 3tanx\, dx} = \cos^3 x$. ------------(0.5M) Hence, the solution is $t\cos^3 x = \int 3\sin x\cos^5 x\ dx + C$ $=> y^3\cos^3 x = -\frac{1}{2}\cos^6 x + C$-----------------(1M)	***3***

17	Using Gauss Seidel method with initial approximation $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$, solve the following system of equations $-3x_1 + 22x_2 + 2x_3 = 47;$ $45x_1 + 2x_2 + 3x_3 = 58;$ $5x_1 + x_2 + 20x_3 = 67.$ Carry out 2 iterations up to 3 decimal places. Soln: $x_1 = \frac{1}{45}(58 - 2x_2 - 3x_3)$ $x_2 = \frac{1}{22}(47 + 3x_1 - 2x_3)$ $x_3 = \frac{1}{20}(67 - 5x_1 - x_2)$ --------(1M) <table><tr><td>Iterations</td><td>x_1</td><td>x_2</td><td>x_3</td></tr><tr><td>1</td><td>1.289</td><td>2.312</td><td>2.912</td></tr><tr><td>2</td><td>0.992</td><td>2.007</td><td>3.001</td></tr></table>------------------(1M)	***2***
18	Solve the following differential equation $(3x^2 tany - cosx)dx + x^3 sec^2 y dy = 0$ Soln: $(3x^2 tany - cosx)dx + x^3 \sec^2 y dy = 0$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3x^2 \sec^2 y$ ------(1M) Soln is $\int 3x^2 tany - cosx dx = c$ $x^3 tany - sinx = c$ ------ (1M)	***2***
19	Using Rayleigh power method, find the numerically largest eigenvalue and the corresponding eigenvector of $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ using the initial vector $X^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Carry out 2 iterations up to three decimal place accuracy. Soln: $AX^{(0)} = 6\begin{pmatrix} 1 \\ 0 \\ 0.667 \end{pmatrix}$ ------------(0.5M) $AX^{(1)} = 7.333\begin{pmatrix} 1 \\ -0.363 \\ 0.545 \end{pmatrix}$ -------(0.5M) Largest eigenvalue is 7.333 and the corresponding eigenvector is $\begin{pmatrix} 1 \\ -0.363 \\ 0.545 \end{pmatrix}$. -------------(1M)	***2***