I Semester BTech (Chemistry Cycle) Mid term Examination- September 2023	
Date: 30 September 2023 Time : :2:45 pm to 4:45 pm Marks : 30	
Course name: Engineering Mathematics I	**Course code: MAT 1171**

Q. No.	Description	Marks	COs	BL
1	The integrating factor in the differential equation, $\frac{dy}{dx} - \frac{y}{1+x} = (1+x)e^x$ is 1. $\log(1+x)$ 2. $-\log(1+x)$ 3. $**\frac{1}{1+x}$ 4. $\frac{-1}{1+x}$	**0.5**	**1**	**2**
2	The solution of $y(2xy + e^x)dx = e^x dy$ is ________. 1. $x + e^x = c$ 2. $x + \frac{e^x}{y} = c$ 3. ** $x^2 + \frac{e^x}{y} = c$ 4. $2x + \frac{e^x}{y} = c$	**0.5**	**1**	**2**
3	The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 6 & -2 \\ 1 & 2 & 4 \end{bmatrix}$ is __ 1. 0 2. 1 3. **2 4. 3	**0.5**	**4**	**2**
4	The system of linear equations $AX = B$ is consistent if, 1. $**Rank\ (A) = Rank([A:B])$ 2. $Rank(A) =$ number of unknowns 3. $Rank([A:B]) =$ number of unknowns 4. $Rank(A) \neq Rank([A:B])$	**0.5**	**4**	**2**
5	Using Gauss-Jacobi method, the value of z in the second iteration from the system of equations $20x + y - 2z = 17$ $3x + 20y - z = -18$ $2x - 3y + 20z = 25$ by taking initial approximation as $x = y = z = 0$ is 1. $z = 0$ 2. $z = 1.25$ 3. $z = 1$ 4. $**z = 1.03$	**0.5**	**4**	**3**
6	Using Rayleigh Power method, the first approximation to largest eigen value of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking the initial eigen vector as $[1 \quad 1 \quad 1]^T$ is 1. 1	**0.5**	**4**	**3**

	2. 4 3. **6 4. 3			
7	The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 0 \\ -1 & -6 \end{bmatrix}$ are 1. **2, -6 2. $-2, 6$ 3. -2, -6 4. 2, 6	**0.5**	**4**	**2**
8	If $A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, then the eigenvalues of A^2 are ______. 1. $2, 3$ 2. $-\frac{1}{2}, -\frac{1}{3}$ 3. $-2, -3$ 4. **$4, 9$	**0.5**	**4**	**2**
9	Which of the following set of vectors form a basis for $\mathbb{R}^2$? 1. $\{(1,0,), (1,1,), (1,2)\}$ 2. **$\{(1,3), (2,-1)\}$ 3. $\{(1,1), (0,0)\}$ 4. $\{(1,-2), (-2,4)\}$	**0.5**	**5**	**2**
10	Which **ONE** of the following is **NOT** a subspace of $V = \mathbb{E}^2 \ or\ \mathbb{R}^2$? 1. The line $x + y = 0$ 2. The line $x - y = 0$ 3. **The line $x + y = 4$. 4. The line $y = 2x$.	**0.5**	**5**	**2**
11	**Show that the set $B = \{(1,1,1), (2,1,0), (5,1,3)\}$ is linearly independent in $\mathbb{R}^3$. Apply Gram-Schmidt orthogonalization process to the vectors in B to determine an orthonormal basis of $\mathbb{R}^3$.** **Ans:** Proving B is linearly independent ……… 1 M $a_1 = (1,1,1), a_2 = (2,1,0), a_3 = (5,1,3)$ $u_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ———— 1 M $v_1 = a_2 - (a_2.u_1)u_1 = (1,0,-1)$ $u_2 = \left(\frac{1}{\sqrt{2}}, 0 - \frac{1}{\sqrt{2}}\right)$ ———— 1 M $v_3 = a_3 - (a_3.u_1)u_1 - (a_3.u_2)u_2 = (1,-2,1)$ $u_3 = \left(\frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{6}}\right)$ ———— 1 M Required orthonormal basis is $\{u_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), u_2 = \left(\frac{1}{\sqrt{2}}, 0 - \frac{1}{\sqrt{2}}\right), u_3 = \left(\frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{6}}\right)\}$	**4**	**5**	**3**

12	**Find the eigen values and any two of the eigen vectors of the matrix** $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Ans: Consider $\lvert A - \lambda I \rvert = 0$ $\Rightarrow (2-\lambda)(-5-\lambda)(3-\lambda) + 6(3-\lambda) = -\lambda^3 + 13\lambda - 12 = 0$ Characteristic equation is $-\lambda^3 + 13\lambda - 12 = 0$. Roots of the characteristic equation is 1,3,-4. Eigenvalues are -4, 3,1 2M For $\lambda = 1$, Consider $AX = X$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Solving we get; $x = 3y \text{ and } z = 0$. Let $y = k$, then $x = 3k$. The eigenvector corresponding to $\lambda = 1$ is $X = \begin{bmatrix} 3k \\ k \\ 0 \end{bmatrix}, k \neq 0$. 1M For $\lambda = -4$, Consider $AX = -4X$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Solving we get; $2x = y \text{ and } z = 0$. Let $y = k$, then $x = \frac{k}{2}$. The eigenvector corresponding to $\lambda = -4$ is $X = \begin{bmatrix} \frac{k}{2} \\ k \\ 0 \end{bmatrix}, k \neq 0$. 1M For $\lambda = 3$, Consider $AX = 3X$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Solving we get; $x = y = 0 \text{ and } z = k$. The eigenvector corresponding to $\lambda = 3$ is $X = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}, k \neq 0$.	4	4	3
13	**Prove that a maximal linearly independent set of vectors in a vector space *V* form a basis for *V*.** Ans: Let $S = \{v_1, v_2, ..., v_n\}$ be a maximal linearly independent set. In order to prove S is a basis, it suffices to prove S spans V. Take $v \in V$. Suppose $\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n + \alpha v = 0$. If $\alpha = 0$, then since $v_1, v_2, ..., v_n$ are linearly independent, we get $\alpha_i = 0$ for all $1 \leq i \leq n$. This means $v, v_1, v_2, ..., v_n$ are (which are n + 1, in number) linearly independent, a contradiction to the maximality of n. 2M Therefore, $\alpha \neq 0$. Now $\alpha v = (-\alpha_1 v_1) + (-\alpha_2 v_2) + ... + (-\alpha_n v_n)$. This implies $v = (-\alpha_1 \alpha^{-1}) v_1 + (-\alpha_2 \alpha^{-1}) v_2 + ... + (\alpha_n \alpha^{-1}) v_n$. Therefore S spans V. Hence S is a basis. 1M	3	5	4
14	**Check whether the set of vectors** $B = \{(4,0,3), (0,4,2), (5,2,4)\}$ **form a basis for** $\mathbb{R}^3$ **or not. If so, then express the vector** $(1,2,2)$ **as a linear combination of the basis elements.** Ans: Let $v_1 = (4,0,3), v_2 = (0,4,2), v_3 = (5,2,4)$ with	3	5	3

	$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$ then $4\alpha_1 + 0\alpha_2 + 5\alpha_3 = 0$ $0\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0$ $3\alpha_1 + 2\alpha_2 + 4\alpha_3 = 0$ $\begin{vmatrix} 4 & 0 & 5 \\ 0 & 4 & 2 \\ 3 & 2 & 4 \end{vmatrix} = -12 \neq 0$, therefore B is linearly independent. —— 1M In an n −dimensional vector space V, any linearly independent set S of n vectors form a basis for V. So, B forms a basis for $\mathbb{R}^3$. Let $(1,2,3) = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ then $4\alpha_1 + 0\alpha_2 + 5\alpha_3 = 1$ $0\alpha_1 + 4\alpha_2 + 2\alpha_3 = 2$ $3\alpha_1 + 2\alpha_2 + 4\alpha_3 = 2$ implies $\alpha_1 = \frac{2}{3}$; $\alpha_2 = \frac{2}{3}$; $\alpha_3 = -\frac{1}{3}$ —— 1M (1, 2, 2) can be represented in terms of basis vectors as $(1,2,2) = \frac{2}{3} v_1 + \frac{2}{3} v_2 - \frac{1}{3} v_3$ —— $\frac{1}{2}$ M			
15	**Solve the differential equation** $\boldsymbol{y\, e^{xy}\, dx + (x\, e^{xy} + 2y)\, dy = 0}$ Solution: $M = y\, e^{xy}$ $N = x\, e^{xy} + 2y$ $M_y = e^{xy} + xy\, e^{xy}$ $N_x = e^{xy} + xy\, e^{xy}$ 2M ∴ equation is exact. Solution is, $\int_{(y\ \text{con})} y\, e^{xy}\, dx + \int 2y\, dy = c$ i.e., $e^{xy} + y^2 = c$ 1M	**3**	**1**	**3**
16	**Using Gauss elimination method, test the consistency and solve the system of linear equations** $\boldsymbol{2x + 5y + 2z - 3w = 3}$ $\boldsymbol{3x + 6y + 5z + 2w = 2}$ $\boldsymbol{4x + 5y + 14z + 14w = 11}$ $\boldsymbol{5x + 10y + 8z + 4w = 4}$	**3**	**4**	**3**

	$(A:B) = \left(\begin{array}{cccc|c} 2 & 5 & 2 & -3 & 3 \\ 3 & 6 & 5 & 2 & 2 \\ 4 & 5 & 14 & 14 & 11 \\ 5 & 10 & 8 & 4 & 4 \end{array}\right)$ $= \left(\begin{array}{cccc|c} 2 & 5 & 2 & -3 & 3 \\ 0 & -3 & 4 & 13 & -5 \\ 0 & -5 & 10 & 20 & 5 \\ 0 & -5 & 6 & 23 & -7 \end{array}\right)$ $R_2 \to 2R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to 2R_4 - 5R_1$ — $\frac{1}{2}$ M $= \left(\begin{array}{cccc|c} 2 & 5 & 2 & -3 & 3 \\ 0 & -3 & 4 & 13 & -5 \\ 0 & 0 & 10 & -5 & 40 \\ 0 & 0 & -2 & 4 & 4 \end{array}\right)$ $R_3 \to 3R_3 - 5R_2$, $R_4 \to 3R_4 - 5R_2$ — $\frac{1}{2}$ M $= \left(\begin{array}{cccc|c} 2 & 5 & 2 & -3 & 3 \\ 0 & -3 & 4 & 13 & -5 \\ 0 & 0 & 2 & -1 & 8 \\ 0 & 0 & 1 & -2 & -2 \end{array}\right)$ $R_3 \to \frac{R_3}{5}$, $R_4 \to \frac{R_4}{-2}$ — $\frac{1}{2}$ M $= \left(\begin{array}{cccc|c} 2 & 5 & 2 & -3 & 3 \\ 0 & -3 & 4 & 13 & -5 \\ 0 & 0 & 2 & -1 & 8 \\ 0 & 0 & 0 & -3 & -12 \end{array}\right)$ $R_4 \to 2R_4 - R_3$ — $\frac{1}{2}$ M The reduced system is $2x + 5y + 2z - 3w = 3$ — (1) $-3y + 4z + 13w = -5$ — (2) $2z - w = 8$ — (3) $-3w = -12$ — (4) On solving eq^ns from (1) to (4) we get, $x = -66,\ y = 22,\ z = 6,\ w = 4$ — 1 M			
17	**Using Gauss-Seidel method to find the approximate solution of the system of linear equations** $6x_1 + x_2 - 3x_3 = -5$ $4x_1 - 8x_2 - x_3 + 2x_4 = 15$ $-x_1 + 7x_3 + 2x_4 = 18$ $-5x_3 + 8x_4 = -23$ **by taking the initial approximation as $x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0$. Carry out 3 iterations and correct to 4 decimal places.** For checking diagonal dominance conditions — 0.5M Re-writing the equations and starting with $[1 \quad 0 \quad 1 \quad 0]$ as the initial guess to the solution, After first iteration the solution is $[-0.3333 \quad -2.1667 \quad 2.5238 \quad -1.2976]$ ——— 1 M After the second iteration the solution is $[0.7897 \quad -2.1200 \quad 3.0550 \quad -0.9656]$ ——— 1 M After the third iteration the solution is $[1.0475 \quad -1.9745 \quad 2.9970 \quad -1.0019]$. ——— $\frac{1}{2}$ M	3	4	3
18	**Solve $\frac{d^3y}{dt^3} - 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} - 3y = 0$.** Ans: Auxillary equation is , $m^3 - 5m^2 + 7m - 3 = 0$ $(m-1)(m^2 - 4m + 3) = 0$ $m = 1, m = 3,1$ — 1 M C.F.= $(c_1 + c_2 t)e^t + c_3 e^{3t}$ ——— 1 M	2	1	3