I Semester BTech (Physics Cycle) Mid term Examination- September 2023	
Date: 30 September 2023 Time : :8:00 am to 10:00 am Marks : 30	
Course name: Engineering Mathematics I	Course code: MAT 1171

Q. No.	Description	Marks	COs	BL
1	The integrating factor of $\frac{dx}{dy} - xy = 1$ is 1. $e^{\frac{y^2}{2}}$ 2. **$e^{-\frac{y^2}{2}}$ 3. e^{y^2} 4. e^{-y^2}	0.5	1	2
2	The general solution of the differential equation $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ is, 1. $e^{xy} = c$ 2. $y^2(1 + e^{xy}) = c$ 3. $2e^{xy} + y^2 = c$ 4. ** $e^{xy} + y^2 = c$	0.5	1	2
3	The values of r and s for which the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$ has rank 2 is 1. $r = 2, s = 2$ 2. ** $r = 2, s = 1$ 3. $r = -2, s = 1$ 4. Cannot have rank 2	0.5	4	2
4	Which of the following statements is incorrect? 1. Rank of an identity matrix of order n is n. 2. Rank of a matrix of order $m \times n$ is less than or equal to minimum of m and n. 3. **Rank of a null matrix is 1. 4. A matrix and its transpose are having the same rank.	0.5	4	3
5	Using Gauss-Jacobi method, the first approximation to solution for following system of equation $10x + y - z = 11.19$ $x + 10y + z = 28.08$ $-x + y + 10z = 35.61$ with the initial approximation $x_0 = y_0 = z_0 = 0$ is 1. **$x = 1.119,\ y = 2.808,\ z = 3.561$ 2. $x = 1,\ y = -1,\ z = 1$ 3. $x = 1.19,\ y = 2.34,\ z = 3.39$ 4. $x = 1.23,\ y = 2.34,\ z = 3.45$	0.5	4	3

6	The largest eigen value in the second iteration of power method for the following matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigen vector $[1, 0, 0]^T$ is 1. 2 2. 1.5 3. **2.5 4. 2.10	**0.5**	**4**	**2**
7	The eigenvalues of a matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ are 1. **2, 4 2. 2, 2 3. 2, -4 4. $-2, 4$	**0.5**	**4**	**3**
8	If the eigenvalues of a 2×2 matrix A are $\frac{1}{2}$ and $-\frac{1}{2}$ then the determinant of A is ? 1. 0 2. $\frac{1}{4}$ 3. **$-\frac{1}{4}$ 4. 1	**0.5**	**4**	**2**
9	Which of the following is a subspace of the vector space $V = \mathbb{E}^2$ *or* $\mathbb{R}^2$? 1. **The line $y = x$. 2. The line $x = 1$. 3. The line $x + y = 1$ 4. The first quadrant of $\mathbb{R}^2$.	**0.5**	**5**	**2**
10	Which of the following set of vectors in $\mathbb{R}^3$ is orthogonal? 1. **$S = \{(1, 0, -1), (1, \sqrt{2}, 1), (1, -\sqrt{2}, 1)\}$. 2. $S = \{(1,1,-1), (2,3,1), (-1,0,1)\}$. 3. $S = \{(1,1,1), (1,-1,1), (1,0,0)\}$ 4. $S = \{(1,-1,1), (1,0,-1), (-1,-1,0)\}$.	**0.5**	**5**	**3**
11	**Show that the set S=** $\{(\mathbf{1}, -\mathbf{1}, \mathbf{1}), (\mathbf{1}, \mathbf{0}, \mathbf{1}), (\mathbf{1}, \mathbf{1}, \mathbf{2})\}$ **is linearly independent in** $\mathbb{R}^3$**. Apply Gram-Schmidt orthogonalization process to the vectors in S to determine an orthonormal basis of** $\mathbb{R}^3$**.** Ans: Proving S is linearly independent ……… 1 M	**4**	**5**	**3**

	$\lvert 1 \quad 1 \quad 2 \rvert$ Let $v_1 = (1,-1,1)$, $v_2 = (1,0,1)$ and $v_3 = (1,1,2)$. $w_1 = v_1$ $w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$ Set $w_2 = (1,2,1)$ $w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2 = \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$ Set $w_3 = (-1,0,1)$. $\therefore \{(1,-1,1), (1,2,1), (-1,0,1)\}$ is an orthogonal basis $\Rightarrow \left\{\frac{1}{\sqrt{3}}(1,-1,1), \frac{1}{\sqrt{6}}(1,2,1), \frac{1}{\sqrt{2}}(-1,0,1)\right\}$ is an orthonormal basis.	1+1+1 M		
12	**Find the eigen values and any two of the eigen vectors of the matrix** $A = \begin{bmatrix} 1 & 1 & -3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. Ans: Characteristic equation is $-\lambda^3 + 7\lambda^2 - 18\lambda + 48 = 0$ ---------(1.5 M) implies $\lambda = 5.31,\ 0.84 \pm 2.88\, i$ are the eigen values---------(1.5 M) Eigen vector corresponds to $\lambda = 5.31$ is $x = \begin{bmatrix} 0.18 \\ 3.77 \\ 1 \end{bmatrix}$---------(1 M)	**4**	**4**	**3**

13	**Prove that a minimal spanning set of vectors in a vector space V form a basis for V.** Let $S = \{v_1, v_2, ..., v_n\}$ be a minimal spanning set. This means $L(S) = V$. In order to prove S is a basis, it suffices to prove S is linearly independent. In a contrary way, suppose that S is not linearly independent. Then there exists v_j (for some j, $1 \le j \le n$) is a linear combination of its preceding ones. That is., $$v_j = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_{j-1} v_{j-1} \text{ for some } \alpha_i \in F,\ 1 \le i \le (j-1).$$ Clearly $L(\{v_1, v_2, ..., v_{j-1}, v_{j+1}, ..., v_n\}) \subseteq L(\{v_i / 1 \le i \le n\}) = L(S)$. On the other hand, take $x \in L(S)$. Then $x = \beta_1 v_1 + \beta_2 v_2 + ... + \beta_n v_n$ for some $\beta_i \in F$, $1 \le i \le n$ $\Rightarrow x = \beta_1 v_1 + ... + \beta_{j-1} v_{j-1} + \beta_j(\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_{j-1} v_{j-1}) + \beta_{j+1} v_{j+1} + ... + \beta_n v_n$ $\Rightarrow x = (\beta_1 + \beta_j \alpha_1) v_1 + ... + (\beta_{j-1} + \beta_j \alpha_{j-1}) v_{j-1} + \beta_{j+1} v_{j+1} + ... + \beta_n v_n$ $$\in L(\{v_1, v_2, ..., v_{j-1}, v_{j+1}, ..., v_n\}).$$ Therefore, $L(\{v_1, v_2, ..., v_{j-1}, v_{j+1}, ..., v_n\}) = L(S) = V$, which is a contradiction to the fact that n is minimum with S spans V. Therefore S is linearly independent. 1.5M 1.5M	3	5	4
14	**Check whether the set of vectors $B = \{(1,1,0), (3,0,1), (5,2,2)\}$ form a basis for $\mathbb{R}^3$ or not. If so, then express the vector $(1,2,3)$ as a linear combination of the basis elements.** Ans: Let $v_1 = (1,1,0), v_2 = (3,0,1), v_3 = (5,2,2)$ with $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$ then $$\alpha_1 + 3\alpha_2 + 5\alpha_3 = 0$$ $$\alpha_1 + 0\alpha_2 + 2\alpha_3 = 0$$ $$0\alpha_1 + \alpha_2 + 2\alpha_3 = 0$$ $\begin{vmatrix} 1 & 3 & 5 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \end{vmatrix} = -3 \neq 0$, therefore B is linearly independent. ---------(1M) In an n −dimensional vector space V, any linearly independent set S of n vectors form a basis for V. So, B forms a basis for $\mathbb{R}^3$. ----------(0.5 M) Let $(1,2,3) = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ then $$\alpha_1 + 3\alpha_2 + 5\alpha_3 = 1$$ $$\alpha_1 + 0\alpha_2 + 2\alpha_3 = 2$$ $$0\alpha_1 + \alpha_2 + 2\alpha_3 = 3$$ implies $\alpha_1 = -\frac{14}{3}; \alpha_2 = -\frac{11}{3};\ \ \alpha_3 = \frac{10}{3}$ ---------(1 M) (1, 2, 3) can be represented in terms of basis vectors as $(1,2,3) = -\frac{14}{3}(1,1,0) - \frac{11}{3}(3,0,1) + \frac{10}{3}(5,2,2)$ ---------(0.5 M)	3	5	3
15	**Solve the differential equation** $$\cos y\, dx + (2x \sin y - \cos^3 y) dy = 0$$	3	1	3

Sol^n $M = \cos y$

$N = 2x\sin y - \cos^3 y$

$$\frac{\partial M}{\partial y} = -\sin y, \quad \frac{\partial N}{\partial x} = 2\sin y$$

Equation is not exact ———— 1M

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -3\frac{\sin y}{\cos y} = f(y)$$

$$I.F = e^{-\int f(y)dy} = \frac{1}{\cos^3 y} \quad \text{———— 1M}$$

Solution,

$$\int \frac{1}{\cos^2 y}dx + \int (-1)\, dy = c$$

$$\frac{x}{\cos^2 y} - y = c$$

Sol^n is, $\boxed{x = (y+c)\cos^2 y}$ ———— 1M

16	**Using Gauss elimination method, test the consistency and solve the system of linear equations.**	3	4	3

$$\boldsymbol{2x_1 + x_2 + 2x_3 + x_4 = 6}$$
$$\boldsymbol{6x_1 - 6x_2 + 6x_3 + 12x_4 = 36}$$
$$\boldsymbol{4x_1 + 3x_2 + 3x_3 - 3x_4 = -1}$$
$$\boldsymbol{2x_1 + 2x_2 - x_3 + x_4 = 10}$$

Ans: Equations are expressed in the matrix form as

$$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 36 \\ -1 \\ 10 \end{bmatrix}$$

Writing the augmented form and using elementary row operations,

$$[A|b] = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix} \quad \text{— } \tfrac{1}{2}\text{M}$$

$R_2 \to R_2 - 3R_1; R_3 \to R_3 - 2R_1; R_4 \to R_4 - R_1$ gives,

$$\begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{bmatrix} \quad \text{— } \tfrac{1}{2}\text{M}$$

$R_2 \to \frac{R_2}{-9}; R_3 \to R_3 - R_2; R_4 \to R_4 - R_2$ gives, $\begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & -3 & 1 & 6 \end{bmatrix}$ — $\frac{1}{2}$M

$R_4 \to R_4 - 3R_3$ gives, $\begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & 0 & 13 & 39 \end{bmatrix}$ — $\frac{1}{2}$M

Solving by the method of back substitution we have,

$$x_4 = 3$$

	$-x_3 - 12 = -11$ implies $x_3 = -1$ $x_2 - 3 = \ -2$ implies $x_2 = 1$ And $2x_1 + 1 - 2 + 3 = 6$ implies $x_1 = 2$. Solution is $x_1 = 2, x_2 - 1, x_3 = -1$, and $x_4 = 3$. — 1M			
17	**Using Gauss-Seidel method to find the approximate solution of the system of linear equations** $$\boldsymbol{-x - y + 10z - 2u = 27}$$ $$\boldsymbol{-x - y - 2z + 10u = -9}$$ $$\boldsymbol{10x - 2y - z - u = 3}$$ $$\boldsymbol{-2x + 10y - z - u = 15}$$ **by taking the initial approximation as $x_0 = y_0 = z_0 = u_0 = 0$. Carry out 3 iterations and correct to 4 decimal places** 1. $x = \frac{1}{10}(3 + 2y + z + u)$ $y = \frac{1}{10}(15 + z + u + 2x)$ $z = \frac{1}{10}(27 + 2u + x + y)$ $u = \frac{1}{10}(-9 + x + y + 2z)$ — $\frac{1}{2}$ M Gauss-Seidel method I iteration $x_1 = 0.3$, $y_1 = 1.56$, $z_1 = 2.886$, $u_1 = -0.1368$ — 1M II iteration $x_2 = 0.8869$, $y_2 = 1.9523$, $z_2 = 2.9566$, $u_2 = -0.0248$ — 1M III iteration $x_3 = 0.9836$, $y_3 = 1.9899$, $z_3 = 2.9924$, $u_3 = -0.0042$. — $\frac{1}{2}$ M $\therefore$ soln is $x = 0.9836$, $y = 1.9899$, $z = 2.9924$, $u = -0.0042$.	3	4	3
18	**Solve $\frac{d^4y}{dx^4} + 6\frac{d^3y}{dx^3} + 9\frac{d^2y}{dx^2} = 0$.** Auxillary equation is , $m^4 + 6m^3 + 9m^2 = 0$ $m^2(m^2 + 6m + 9) = 0$ $m = 0,0, m = -3,-3$ — 1 M C.F.= $(c_1 + c_2t)e^{0t} + (c_3 + c_4t)e^{-3t}$ — 1 M	2	1	3