Question Paper

Exam Date & Time: 03-Jun-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SECOND SEMESTER B.TECH. EXAMINATIONS - MAY/JUNE 2023
SUBJECT: MAT 1271-CHM/ MAT 1271-CHM-B/MAT 1251-B - ENGINEERING MATHEMATICS - II

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Find the extreme values of $f(x,y) = x^3 + y^3 - 3xy$. (4)

1B) Expand $g(x,y) = log(1+x+y)$ up to 3rd degree terms by Maclaurin's series. (3)

1C) Evaluate $\lim_{x \to 0} \frac{x - \tan x}{x^3}$ (3)

2A) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$ orthogonally. (4)

2B) If $u = f(r)$ where $x^2 + y^2 = r^2$ then show that (3)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

2C) Using beta and gamma functions, evaluate $\int_0^\infty \frac{x^2}{(1+x^4)^3}\, dx$ (3)

3A) Using Laplace transform, solve the differential equation $x''(t) + 2x'(t) + x(t) = 3e^{-t}$, $x(0) = 4$, and $x'(0) = 2$. (4)

3B) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2}\, dy\, dx$ by transforming to polar coordinates. (3)

3C) A balloon in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m, find the percentage change in the volume of balloon. (3)

4A) Test for convergence of the series (4)

$$\sum_{n=1}^{\infty} \left(\frac{n^3 + 3}{2n^5 + 1} \right)$$

4B) Using double integrals, find the area included between the cardioids $r = a(1 + cos\theta)$ and $r = a(1 - cos\theta)$. (3)

4C) Find $L^{-1}\left\{\frac{s}{(s-1)(s+4)}\right\}$ (3)

5A) Test the convergence of the series (4)

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots \text{ where } x > 0$$

5B) Using cylindrical polar coordinates, evaluate $\iiint (x^2 + y^2)\, dx\, dy\, dz$ over the region bounded by the paraboloid $x^2 + y^2 = 3z$ and the plane $z = 3$ (3)

5C) Find $L\left\{\frac{1-cos2t}{t}\right\}$ (3)

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