# Question Paper

Exam Date & Time: 11-Jul-2023 (09:30 AM - 12:30 PM)

## MANIPAL ACADEMY OF HIGHER EDUCATION

**SECOND SEMESTER B.TECH. DEGREE EXAMINATIONS - JUNE/JULY 2023**
**SUBJECT: MAT 1271-PHY/MAT 1271-PHY-B - ENGINEERING MATHEMATICS - II**
**(MAKEUP)**

**Marks: 50** **Duration: 180 mins.**

**Answer all the questions.**

1A) Find the maximum and minimum values of (4)
$f(x,y) = x^3 + 3x^2 + 8xy + 4y^2$ .

1B) Evaluate $\lim_{x\to 0}\left(\frac{1-\cos x}{x\sin x}\right)$ (3)

1C) Obtain the Taylor series expansion for $f(x,y) = e^{x+y}$ about the point $(1,1)$ upto third degree terms. (3)

2A) Find the equation of the sphere having the circle (4)
$x^2 + y^2 + z^2 - 3x + 4y - 2z - 5 = 0,\ 5x - 2y + 4z + 7 = 0$
as a great circle.

2B) Using Euler's theorem, show that if (3)
$u = \log\left(\frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}\right)$ then,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}e^u.$$

2C) Using beta and gamma functions, find $\int_0^2 x^2\sqrt{2-x}\ dx$. (3)

3A) Using Laplace transforms, solve the differential equation (4)
$$y'' + 7y' + 12y = 0$$
with initial conditions $y(0) = 0\ and\ y'(0) = 1$.

3B) Change the order of integration and evaluate $\int_{y=0}^{3}\int_{x=0}^{3-y} y\ dx\ dy$ (3)

3C) If $u = F(2x-3y, 3y-4z, 4z-2x)$ then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$ (3)

4A) Test for convergence of the series (4)

$$\sum_{n=1}^{\infty} \frac{3n+1}{n(2n+1)(n+3)}$$

4B) Using double integrals, find the area of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. (3)

4C) Find $L^{-1}\left(\frac{1}{(s-6)(s-5)}\right)$ (3)

5A) Using Ratio test, discuss the nature of the series. (4)

$$1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} \cdots$$

5B) Evaluate $\int_{x=1}^{2} \int_{y=0}^{1} \int_{z=1}^{2} (x + y + z)\, dz\, dy\, dx$ (3)

5C) Find the Laplace transform of $f(t) = e^{-3t} \cos 3t - 3t^{\frac{1}{2}}$ (3)

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