Question Paper

Exam Date & Time: 12-Jul-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SECOND SEMESTER B.TECH. EXAMINATIONS - JUNE/JULY 2023
SUBJECT: MAT 1271-CHM/MAT 1271-CHM-B - ENGINEERING MATHEMATICS - II

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Find the maximum and minimum values of (4)
$f(x,y) = x^2 + 2xy + 2y^2 + 2x + y$.

1B) Evaluate $\lim_{x\to 0}\left(\frac{x-\log(1+x)}{x^2}\right)$ (3)

1C) Expand $f(x,y) = e^x \cos y$ in powers of x and y upto third degree terms. (3)

2A) Find the equation of the sphere having the circle (4)
$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0,\ x + y + z = 3$ as a great circle.

2B) Using Euler's theorem, show that, if $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then (3)
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

2C) Using beta and gamma functions, find $\int_0^1 x^4\sqrt{1-x^2}\ dx$. (3)

3A) Using Laplace transforms, solve the differential equation (4)
$$y'' - 5y' + 6y = 0$$
with initial conditions $y(0) = 0$ *and* $y'(0) = 1$.

3B) Change the order of integration and evaluate $\int_{x=0}^{2}\int_{y=0}^{2-x} xy\ dy\ dx$ (3)

3C) If $u = F(x-y, y-z, z-x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (3)

4A) Test for convergence of the series (4)
$$\sum_{n=1}^{\infty}\frac{2n-1}{n(n+1)(n+2)}$$

4B) Using double integrals, find the area of the region enclosed by the parabola (3)
$y = x^2$ and the line $y = x$.

4C) (3)

Find $L^{-1}\left(\frac{s+1}{(s+5)(s-7)}\right)$

5A) Using Ratio test, discuss the nature of the series. (4)

$$\frac{1}{3}+\frac{2^2}{3^2}+\frac{3^2}{3^3}+\frac{4^2}{3^4}\ldots$$

5B) Evaluate $\int_{x=1}^{2}\int_{y=2}^{3}\int_{z=1}^{3}(x^2y+z)\,dz\,dy\,dx$ (3)

5C) Find the Laplace transform of $f(t)=t\sin 4t+4t^{\frac{5}{2}}$ (3)

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