Exam Date & Time: 13-Mar-2023 (04:15 PM - 05:15 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

ENGINEERING MATHEMATICS - II [MAT 1271]

Marks: 15 **Duration: 60 mins.**

Multiple Choice Questions

Answer all the questions. Section Duration: 20 mins

1) The value of
$$\lim_{x\to 1}\frac{x\log x}{x^2-1}$$
is

(0.5)

1) 2 2) 0 3) $\frac{1}{2}$ 4) -1

Correct option is: 3

2) The value of
$$\lim_{x\to 0}(a^x+x)^{\frac{1}{x}}$$
is

(0.5)

1) $1+\log a$ 2) $\log a$ 3) $\log(1+a)$ 4) ae

Correct option is: 4

3) If $u=x^2+y^2$ where $x=t^2, y=2t$ then the total derivative $\frac{du}{dt}$ is

1)	$2(x+yt)$		2)	$4(x+ty)$		3)	$4(xt+y)$		4)	$2(xt+y)$	

(0.5)

Correct option is: 3

4) The coefficient of x in the Maclaurin's series expansion of $e^{\sin x}$ is equal to

1)	0		2)	1		3)	2		4)	3	

(0.5)

Correct option is: 2

5) Taylor's series expansion of $\frac{1}{x}$ about x = 1 is

1)	$1-(x-1)+(x-1)^2-(x-1)^3+\cdots$		2)	$1+(x-1)+(x-1)^2+(x-1)^3+\cdots$		3)	$1-2(x-1)+3(x-1)^2-4(x-1)^3+\cdots$		4)	$1+2(x-1)+3(x-1)^2+4(x-1)^3+\cdots$	

(0.5)

Correct option is: 1

6) If
$$u=x^3y^2\sin^{-1}\left(\frac{y}{x}\right)$$
then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ =...

(0.5)

1)	$5u$		2)	$20u$		3)	$3u$		4)	$2u$	

Correct option is: 1

7) If the functions $\sin x$ and $\cos x$ satisfy Cauchy's mean value theorem in $\left[-\frac{\pi}{2},0\right]$, then the value of 'c' is

(0.5)

1)	0		2)	$-\frac{\pi}{3}$		3)	$-\frac{\pi}{6}$		4)	$-\frac{\pi}{4}$	

Correct option is: 4

8) If $pv^2 = k$ and the relative errors in p and v are respectively 0.05 and 0.025 then the percentage error in k is

1)	5		2)	7. 5		3)	10		4)	15	

(0.5)

Correct option is: 3

9) If $u = \sin xy + x \log y$ then the value of $\frac{\partial^2 u}{\partial x \partial y}$ at $\left(0, \frac{\pi}{2}\right)$ is

1)	$\frac{\pi+2}{\pi}$		2)	$\frac{2}{\pi}$		3)	$\frac{\pi}{2}$		4)	0	

(0.5)

Correct option is: 1

10) The minimum value of f(x, y) = $x^2 + y^2 + 6x + 12$ is ....

1)	3		2)	1		3)	-3		4)	12	

(0.5)

Correct option is: 1

Descriptive Type Questions

Answer all the questions.

11) If $v = r^m$ and $r = \sqrt{x^2 + y^2}$ then show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = m^2 r^{m-2}$. (2)

12) Find the constants a, b and c so that $\lim_{x \to 0} \frac{ae^x - bcosx + ce^{-x}}{xsinx} = 2$. (2)

13) Obtain Taylor's series expansion of $tan^{-1}\left(\frac{y}{x}\right)$ about (1, 1) up to and including the second degree terms. (3)

14) Using Lagrange's method of undetermined multipliers, find the minimum value of x^2yz^3 subject to the condition $2x + y + 3z = 6$. (3)

-----End-----

Ans 11)

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial r}\frac{\partial r}{\partial x} = m r^{m-1} \cdot \frac{\partial r}{\partial x}$$

$$\frac{\partial r}{\partial x} = \frac{x}{r} \text{ and } \frac{\partial r}{\partial y} = \frac{y}{r}$$

$$\therefore \frac{\partial V}{\partial x} = m x r^{m-2} \text{ and}$$

$$\text{III}^{ly} \frac{\partial V}{\partial y} = m y r^{m-2}$$

1M

$$\frac{\partial^2 V}{\partial x^2} = m\left[x(m-2)r^{m-3} \cdot \frac{\partial r}{\partial x} + r^{m-2}\right]$$

$$= m\left[x^2(m-2)r^{m-4} + r^{m-2}\right]$$

$$\text{III}^{ly} \frac{\partial^2 V}{\partial y^2} = m\left[y^2(m-2)r^{m-4} + r^{m-2}\right]$$

$$\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = m\left[(m-2)r^{m-4}(x^2+y^2) + 2r^{m-2}\right]$$

$$= \underline{\underline{m^2 r^{m-2}}}$$

1M

12) $$\lim_{x\to 0} \frac{ae^{x}-b\cos x+ce^{-x}}{x^2} = 2 \quad \text{——} ①$$

As the denominator is zero for $x=0$ then ① tends to a finite limit iff the numerator also becomes '0' for $x=0$.

ie; $a-b+c=0$ —— ½M

with this LHS of ① is of the form $\left(\frac{0}{0}\right)$.

$$\therefore \lim_{x\to 0} \frac{ae^{x}+b\sin x-ce^{-x}}{2x} = 2 \quad \text{——} \tfrac{1}{2}M$$

Using the similar argument,

$$a-c=0 \Rightarrow a=c$$

$$\therefore \lim_{x\to 0} \frac{ae^{x}+b\cos x+ce^{-x}}{2} = 2$$

½M

$$\Rightarrow a+b+c=4$$
$$a=c$$
$$a-b+c=0$$
$$\Rightarrow a=c=1, \quad b=2$$

½M

13) $f(x,y) = \tan^{-1}(y/x) \Rightarrow f(1,1) = \pi/4$ — ½m

$f_x = \dfrac{-y}{x^2+y^2} \Rightarrow f_x(1,1) = \dfrac{-1}{2}$

$f_y = \dfrac{x}{x^2+y^2} \Rightarrow f_y(1,1) = \dfrac{1}{2}$ — $\frac{1m}{2}$

$f_{xx} = \dfrac{2xy}{(x^2+y^2)^2} \Rightarrow f_{xx}(1,1) = \dfrac{1}{2}$ — ½m

$f_{yx} = \dfrac{y^2-x^2}{(x^2+y^2)^2} \Rightarrow f_{yx}(1,1) = 0$ — ½m

$f_{yy} = \dfrac{-2xy}{(x^2+y^2)^2} \Rightarrow f_{yy}(1,1) = \dfrac{-1}{2}$ — ½m

∴ The req'd expansion is, — ½m

$\tan^{-1}(y/x) = \dfrac{\pi}{4} - \dfrac{(x-1)}{2} + \dfrac{(y-1)}{2} + \dfrac{(x-1)^2}{4} - \dfrac{(y-1)^2}{4} + \cdots$

14) Lagrange's function is: $F(x, y, z, \lambda) = x^2yz^3 + \lambda(2x + y + 3z - 6)$ **----(0.5M)**
For stationary points,

$$\left.\begin{aligned} \frac{\partial F}{\partial x} &= 2xyz^3 + 2\lambda = 0 \\ \frac{\partial F}{\partial y} &= x^2z^3 + \lambda = 0 \\ \frac{\partial F}{\partial z} &= 3x^2yz^2 + 3\lambda = 0 \end{aligned}\right\} \text{----(1M)}$$

$\Rightarrow \lambda = xyz^3 = x^2z^3 = x^2yz^2 \Longrightarrow x = y = z$ **----(0.5M)**
Since, $2x + y + 3z = 6$, we have, $x = y = z = 1$**----(0.5M)**
Stationary point is $(1,1,1)$. Minimum value of is $f = 1$. **----(0.5M)**