II Semester B.Tech (Common to all Branches) Mid Term Examination March 2024

Subject: Engineering Mathematics II (MAT 1271) Date: 18 March 2024 Time: 2:45 pm to 4:45 pm

Type: MCQ

Q1. The stationary point of $f(x, y) = x^3 + y^3 - 3x - 12y + 20 = 0$ is (0.5) **CO2 BL-3**

A. $(1,1)$

B. $(2,1)$

C. $(-1,1)$

D. **$(-1,2)$

Q2. The coefficient of y^2 in the Maclaurin's series expansion of $f(x, y) = e^x \log(1 + y)$ is (0.5) **CO2 BL-3**

A. **$-\frac{1}{2}$

B. -1

C. 1

D. ½

Q3. The circumference of a circle is measured as $28\ cm$ with an error of $0.01\ cm$. The percentage error in the area is (0.5) **CO1 BL-3**

A. **$\frac{1}{14}$

B. $\frac{1}{7}$

C. $\frac{1}{28}$

D. $-\frac{1}{28}$

Q4. The value $\lim\limits_{x \to 0} \frac{3e^x - 2e^{2x} - e^{3x}}{e^x + e^{2x} - 2e^{3x}}$ is (0.5) **CO2 BL-3**

A. **$\frac{4}{3}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Q5. By changing the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ into polar coordinates, we get (0.5) **CO3 BL-3**

A. $\int_0^\infty \int_0^{\frac{\pi}{2}} e^{-r^2} d\theta dr$

B. **$\int_0^\infty \int_0^{\frac{\pi}{2}} e^{-r^2} r d\theta dr$

C. $\int_0^{\infty}\int_0^{\frac{\pi}{2}} e^{-r^2} 2r d\theta dr$

D. $\int_0^{\infty}\int_0^{\frac{\pi}{2}} e^{-r^2} r^2 d\theta dr$

Q6. The value of $\int_0^1\int_0^x e^{\frac{y}{x}}\, dydx$ is (0.5) **CO3 BL-3**

A. **$\frac{e-1}{2}$

B. $e-1$

C. e^{-2}

D. e

Q7. The center and radius of the sphere $x^2+y^2+z^2-2y-4z=11$ are (0.5) **CO1 BL-3**

A. $Center\ (0,-1,-2)$ and $r=4$

B. **$Center\ (0,1,2)$ and $r=4$

C. $Center\ (0,\ 1,-2)$ and $r=4$

D. $Center\ (0,-1,\ 2)$ and $r=4$

Q8. If the functions $f(x)=x^3$ and $g(x)=x^2$ satisfy all the conditions of Cauchy's mean value theorem in $[1,2]$ then the value of c is (0.5) **CO2 BL-3**

A. $\frac{5}{3}$

B. **$\frac{14}{9}$

C. 1

D. 2

Q9. If $x = u + uv$ and $y = uv$ then the Jacobian $J=\frac{\partial(x,y)}{\partial(u,v)}$ is (0.5) **CO2 BL-3**

A. **u
B. $u+2uv$
C. uv
D. v

Q10. The Maclaurin's series expansion of $y = \sin x$ is (0.5) **CO2 BL-3**

A. $x+\frac{x^3}{2!}+\cdots$

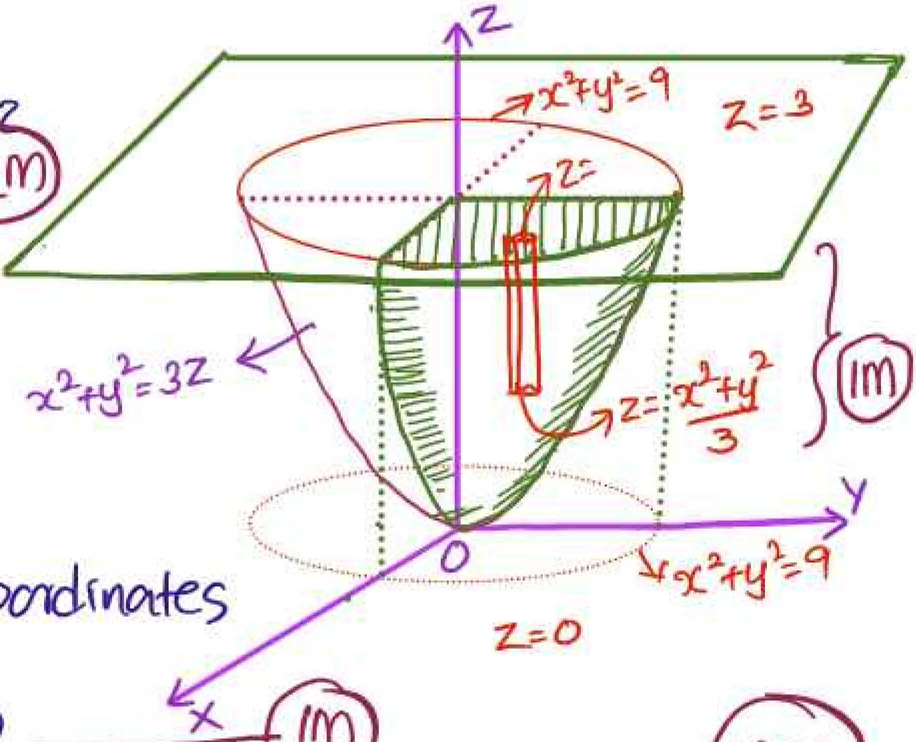
B. **$x-\frac{x^3}{3!}+\cdots$

C. $1+x+\frac{x^3}{3!}+\cdots$

D. $1 - x + \frac{x^3}{3!} + \cdots$

Type: DES

Q11. Using triple integration, find the volume of the paraboloid $x^2 + y^2 = 3z$ cut off by the plane $z = 3$. (4) **CO3 BL-3**



Ans:- Volume $= 4 \times \iiint dx\,dy\,dz$ — (½M)

$$= 4\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{z=\frac{x^2+y^2}{3}}^{3} dz\,dy\,dx \quad \text{— (1M)}$$

$$= 4\int_0^3 \int_0^{\sqrt{9-y^2}} \left(\frac{x^2+y^2}{3}\right) dy\,dx$$

By changing to polar coordinates

$$= 4\int_{\theta=0}^{\pi/2} \int_{r=0}^{3} \frac{r^2}{3}\, r\,dr\,d\theta \quad \text{— (1M)}$$

$$= 4\int_{\theta=0}^{\pi/2} \left(\frac{r^4}{12}\right)_0^3 d\theta = 27\int_{\theta=0}^{\pi/2} d\theta = \frac{27\pi}{2} \text{ cubic units} \quad \text{— (½M)}$$

Q12. Using Lagrange's method of undetermined multipliers, find the maximum and minimum distances of the point (1, 2, 3) from the sphere $x^2 + y^2 + z^2 = 56$. (4) **CO2 BL-3**

Ans:. Let $P(x,y,z)$ be any point on the sphere and $A(1,2,3)$. Then $AP^2 = (x-1)^2 + (y-2)^2 + (z-3)^2 = f(x,y,z)$ — (1M)

$\phi : x^2+y^2+z^2-56 = 0$

Lagrange funct, $L = f + \lambda\phi$ — (½M)

$\frac{\partial L}{\partial x} = 0 \Rightarrow x-1 = -\lambda x$ $\quad \frac{\partial L}{\partial y} = 0 \Rightarrow y-2 = -\lambda y$ — (1M)

$\frac{\partial L}{\partial z} = 0 \Rightarrow z-3 = -\lambda z$ $\quad \Rightarrow x = \frac{1}{1+\lambda},\ y = \frac{2}{1+\lambda},\ z = \frac{3}{1+\lambda}$

$x^2+y^2+z^2 = 56 \Rightarrow (1+\lambda)^2 = \frac{1}{4} \Rightarrow 1+\lambda = \pm\frac{1}{2}$ — (½M)

$\therefore$ Stationary points are $P_1(2,4,6)$, $P_2(-2,-4,-6)$ — (½M)

$\Rightarrow AP_1 = \sqrt{14} = 3.7$ (Min), $AP_2 = \sqrt{126} = 11.22$ (Max) — ½M

Q13. Expand $f(x,y) = \sin(xy)$ in powers of $(x-1)$ and $(y-\frac{\pi}{2})$ up to second degree terms. (3) **CO2 BL-3**

Ans:

$$\left.\begin{aligned}
f(x,y) &= \sin(xy) \Rightarrow f(1,\pi/2) = 1\\
f_x &= y\cos(xy) \Rightarrow f_x(1,\pi/2) = 0\\
f_y &= x\cos(xy) \Rightarrow f_y(1,\pi/2) = 0\\
f_{xx} &= -y^2\sin(xy) \Rightarrow f_{xx}(1,\pi/2) = -\pi^2/4\\
f_{yy} &= -x^2\sin(xy) \Rightarrow f_{yy}(1,\pi/2) = -1
\end{aligned}\right\} \text{2M}$$

$$f_{xy} = -xy\sin(xy) + \cos(xy) \Rightarrow f_{xy}(1,\pi/2) = -\pi/2$$

$\therefore$ Req'd expansion is,

$$\sin(xy) = 1 - \frac{\pi^2}{8}(x-1)^2 - \frac{\pi}{2}(x-1)(y-\pi/2) - \frac{1}{2}(y-\pi/2)^2 + \cdots$$ — 1M

Q14. Find the equation of the sphere which passes through the circle

$$S: x^2+y^2+z^2-2x-3y+4z+8=0 \;; U: x-2y+z-8=0$$

and has its center on the plane $4x-5y-z-3=0$. (3) **CO1 BL-3**

Ans:. Eq^n of the req'd sphere : $S + KU = 0$ — $1\frac{1}{2}$M

$$\Rightarrow x^2+y^2+z^2+x(-2+K)+y(-3-2K)+z(4+K)+8-8K=0 \quad (*)$$

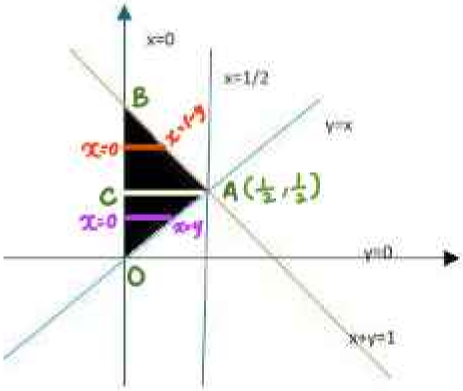
Centre of (*) is, $\left(\frac{2-K}{2}, \frac{3+2K}{2}, \frac{-4-K}{2}\right)$ — ½M

This centre lies on $4x-5y-z-3=0$

$$\Rightarrow K = \frac{-9}{13}$$ — ½M

Req'd eq^n: $13(x^2+y^2+z^2) - 35x - 21y + 43z + 32 = 0$ —— ½M

Q15. The region of evaluation of the integral $I = \int_0^{1/2}\int_x^{1-x} xy\,\mathrm{dy}\,\mathrm{dx}$ is shaded in the following diagram.



Change the order of integration and hence evaluate it. (3) **CO3 BL-3**

Ans:

$$I = \int_0^{1/2}\int_0^{y} xy\,dx\,dy + \int_{1/2}^{1}\int_0^{1-y} xy\,dx\,dy \quad \text{—— 1M}$$

$$= \frac{1}{2}\int_0^{1/2} y^3\,dy + \frac{1}{2}\int_{1/2}^{1} (1-y)^2 y\,dy \quad \text{—— 1M}$$

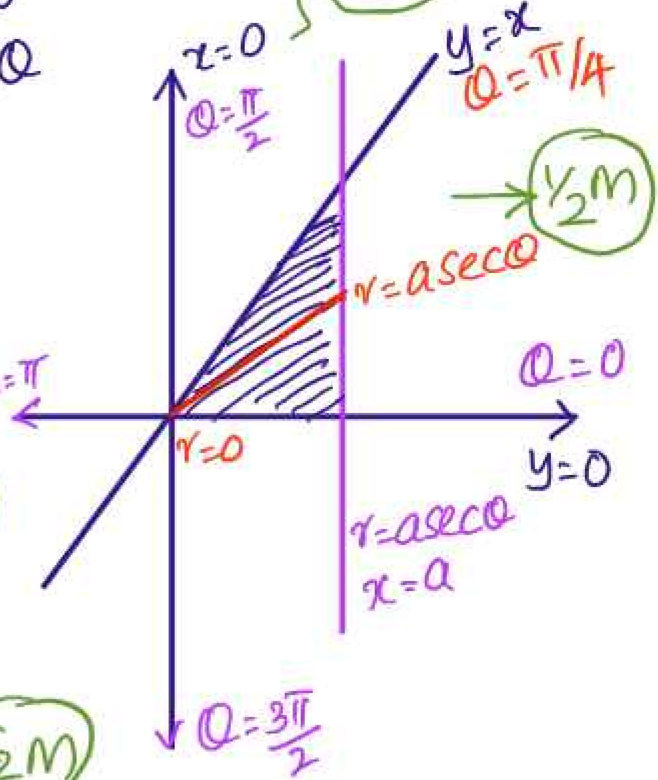
$$= \frac{1}{2}\left(\frac{y^4}{4}\right)_0^{1/2} + \frac{1}{2}\left(\frac{y^2}{2} + \frac{y^4}{4} - \frac{2y^3}{3}\right)_{1/2}^{1} \quad \text{—— ½M}$$

$$= \frac{1}{48} \quad \text{—— ½M}$$

Q16. Evaluate $\int_0^a \int_y^a y\sqrt{x^2+y^2}\,\mathrm{dx}\,\mathrm{dy}$ where $a > 0$ by changing to polar coordinates. (3)

CO3 BL-3

Ans: put $x = r\cos\theta$, $y = r\sin\theta$ (1/2M)
$dx\,dy = r\,dr\,d\theta$



Let $I = \int_0^a \int_y^a y\sqrt{x^2+y^2}\,dx\,dy$

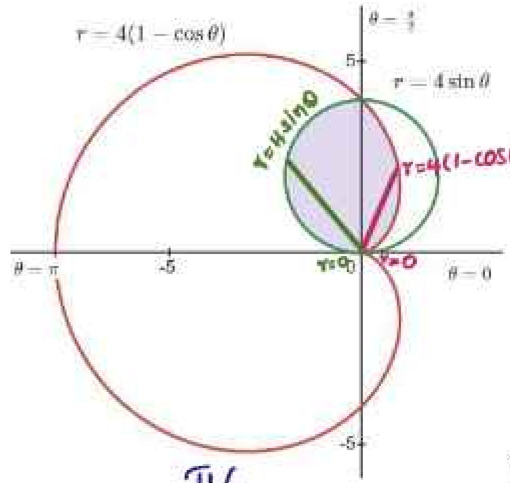
$= \int_{\theta=0}^{\pi/4} \int_{r=0}^{a\sec\theta} r\sin\theta \cdot r \cdot r\,dr\,d\theta$ — (1M)

$= \int_{\theta=0}^{\pi/4} \sin\theta \left(\frac{r^4}{4}\right)_0^{a\sec\theta} d\theta$ — (1/2M)

$= \frac{a^4}{4} \int_{\theta=0}^{\pi/4} \sin\theta \sec^4\theta\,d\theta$

$= \frac{a^4}{4} \times \frac{(2\sqrt{2}-1)}{3} = \underline{\underline{\frac{a^4(2\sqrt{2}-1)}{12}}}$ (1/2M)

Q17. Using double integrals, find the area of the shaded region in the given diagram, bounded by the curves $r = 4(1-\cos\theta)$ and $r = 4\sin\theta$.



(3) **CO3 BL-3**

Ans:- $$\text{Area} = \int_{\theta=0}^{\pi/2}\int_{r=0}^{r=4(1-\cos\theta)} r\,dr\,d\theta + \int_{\theta=\frac{\pi}{2}}^{\pi}\int_{r=0}^{4\sin\theta} r\,dr\,d\theta \quad \text{(1M)}$$

$$= \int_{\theta=0}^{\pi/2} 8(1-\cos\theta)^2 d\theta + \int_{\theta=\pi/2}^{\pi} 8\sin^2\theta\, d\theta \quad \text{(1M)}$$

$$= 8\int_{\theta=0}^{\pi/2}(1-2\cos\theta+\cos^2\theta)d\theta + 8\int_{\theta=\pi/2}^{\pi}\left(1-\frac{\cos 2\theta}{2}\right)d\theta$$

$$= 8\left(\theta-2\sin\theta\right)_0^{\frac{\pi}{2}} + 8\cdot\frac{1}{2}\times\frac{\pi}{2} + 4\left(\theta-\frac{\sin 2\theta}{2}\right)_{\pi/2}^{\pi} \quad \text{(1/2M)}$$

$$= 8\left(\frac{\pi}{2}-2\right) + 2\pi + 4(\pi/2)$$

$$= \underline{8\pi - 16} \text{ sq. units} \quad \text{(1/2M)}$$

Q18. If $\log u = \frac{x^3+y^3}{3x+4y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\log u$. (2) **CO2 BL-3**

Ans: $\because$ $\log u$ is a homogenous funct. in x & y of deg. 2. — (1/2M)

$\therefore$ By Euler's thm, $x\frac{\partial}{\partial x}(\log u) + y\frac{\partial}{\partial y}(\log u) = 2\log u$ — (1/2M)

$\Rightarrow x \cdot \frac{1}{u}\frac{\partial u}{\partial x} + y\frac{1}{u}\frac{\partial u}{\partial y} = 2\log u$ — (1/2M)

$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\log u$ — (1/2M)